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Application of modern approach of Caputo-Fabrizio fractional derivative to MHD second grade fluid through oscillating porous plate with heat and mass transfer

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ABSTRACT

In this research paper, we analyze the flow characteristics of magnetohydrodynamic second grade fluid with heat and mass transfer embedded in porous medium. The modeling of partial differential equations governs the flow have been established with modern approach of Caputo-Fabrizio fractional operator $^{CF}\left(\frac{\partial^{\delta}}{\partial t^{\delta}}\right)$. The partial differential equations of non-integer order derivatives have been solved by invoking Laplace and Fourier sine transforms. The new analytic solutions for temperature, concentration and velocity are investigated and expressed in terms of simple elementary functions. The corresponding general solutions have been particularized with and without magnetic field and porous medium for the classical Newtonian and second grade fluids as the limiting cases of our general results. The effects of the embedded physical and geometric parameters have been depicted through graphs for velocity, temperature and concentration respectively. The graphical results show several physical discrepancies and analogies on the fluid flow. Finally, our results suggest that increasing the Grashof number, heat transfer due to convection facilitates the flow velocity profile and an opposite trend is observed in thermal Grashof number as well.

1. Introduction

It is well established fact that flow of non-Newtonian liquids has great significance and capability than flow of Newtonian liquids in practical applications and technological development. The mathematicians, engineers, and numerical analysts have diverted their attention towards varied challenges of non-Newtonian liquids, and developing appropriate analytical and numerical solutions via different mathematical and experimental strategies. Due to highly nonlinear nature of the governing equations from the flow of non-Newtonian liquids, solution is still narrowed down extensively. In order to understand the characteristics and complexities of flow of non-Newtonian liquids, there is no single model which can completely characterize all the properties of non-Newtonian liquids. In brevity, the differential-type non-Newtonian liquids for instance third grade fluids and second grade fluids have achieved the significant attention of researchers. Some important applications of non-Newtonian liquids include, flow of mercury amalgams, flow of plasma, flow of liquid metals and alloys, flow of blood, flow of nuclear fuel slurries, chyme movement in the intestine, lubrications with heavy oils and greases, polymer solutions, food mixing, paint and several others (Dunn and Fosdick, 1974; Dunn and Rajagopal, 1995; Asghar et al., 2004; Vieru et al., 2008; Abro and Solangi, 2017; Fetecau and Fetecau, 2005; Nadeem, 2006; Laghari et al., 2017; Abro, 2016). In this connection, mixed convection flow has played a significant role in the development and applications in industry and technology. From industrial and technological point of view, we affix here a few applications for instance, heat exchangers placed in a low-velocity environment, solar central receivers exposed to wind currents, nuclear reactors cooled during emergency shutdown, electronic devices cooled by fans, rotating heat exchanger, geothermal reservoirs, containers of nuclear waste disposal, spin-stabilized missiles and many others as well (Li et al., 2011). Nadeem and Saleem (2014) observed rotating second grade fluid for unsteady mixed convection flow in a rotating cone. They presented two cases namely prescribed heat flux (PHF) and prescribed wall temperature (PWT) via analytical approach of the homotopy analysis

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method (HAM). Khan et al. (2017a) analyzed heat and mass transfer flow for MHD thin-film second-grade fluid for the influences of thermal radiation and thermophoresis. They converted highly nonlinear coupled differential equations for the velocity field, temperature distribution and mass concentration of thin-film second-grade fluid flow by invoking appropriate similarity transformations and obtained solutions by implementing homotopy analysis method (HAM). Khan et al. (2017b) investigated Jeffery-Hamel flow of second-grade fluid for viscous dissipation, Dufour and Soret effects for stretchable walls. They analyzed Jeffery-Hamel flow of second-grade fluid using analytical and numerical approaches namely homotopy analysis method (HAM) and Runge-Kutta scheme respectively. Labropulu and Li (2016) worked on stagnation point flow of second-grade fluid on the plate. They transferred the governing partial differential equations into a system of ordinary differential equations and solved numerically using a shooting method. Their main significance was to check the effects of the Weissenberg number on the velocity near the wall. Hayat et al. (2016) explored the impacts of MHD second grade fluid flow between two parallel disks. They investigated heat transfer analysis due to convective boundary condition and thermal radiation and obtained convergent solutions by applying homotopic approach. In order to disclose the physical aspects of this study, Skin friction coefficient and Nusselt number were also analyzed numerically. Bataineh et al. (2016) presented approximate solution for the heat transfer problem of second-grade fluid in a channel embedded with porosity based on the method of Bernstein polynomials. For the sake of physics of the heat transfer problem of second grade fluid, they applied the residual correction procedure for the estimation of the absolute error. They also compared analysis and results via homotopy analysis method and Runge-Kutta fourth order method. Hayat et al. (2017) examined the characteristics of temperature dependent thermal conductivity and thermal stratification for stretched flow of second grade liquid. They emphasized on the salient features of thermal relaxation time that revealed that temperature distribution enhanced via larger variable thermal conductivity parameter. Gul et al. (2015) obtained analytical solutions of second grade fluid over a vertical oscillating belt by invoking Adomian decomposition method (ADM). They emphasized thin film flow of second grade fluid for the comparative analysis of absolute error between Adomian decomposition method (ADM) and Optimal asymptotic method (OHAM). Shah and Khan (2016) investigated an interesting analysis for the thermal analysis of second grade fluid using modern approach of fractional calculus. They invoked Caputo–Fabrizio fractional derivatives approach on second grade fluid over an infinite oscillating plate. They analyzed temperature differences between the plate and the fluid and concluded that the heat transfer is caused by the buoyancy force. They also investigated the closed form solutions for temperature distribution and velocity profile and presented via graphical illustrations. Arshad et al. (2017) presented significant study of heat and mass transfer of second grade fluid via comparative analysis of Caputo–Fabrizio and Atanagna-Baleanu fractional derivatives. They nondimensionalized the governing partial differential equations of mass concentration, temperature distribution and velocity field and solved separately for comparison via Caputo–Fabrizio and Atanagna-Baleanu fractional derivatives. Off course the studies on heat and mass transfer of second grade fluid (Khan et al., 2017a; 2017b; Labropulu and Li, 2016; Hayat et al., 2016) can continue but we end here by citing few recent references under different geometries and approaches (Ali et al., 2012; Gómez-Aguilar et al., 2016; Abro et al., 2018a; 2018b; Jordan, 2017; Khan et al., 2018; Ahmed and Khan, 2018; Mishra et al., 2018; Hussanan et al., 2018). Motivating by above discussions especially from modern fractional approaches and methodology of the solutions, our aim is to analyze the flow characteristics of magnetohydrodynamic second grade fluid with heat and mass transfer embedded in porous medium. The modeling of partial differential equations governs the flow has been established with modern approach of Caputo-Fabrizio fractional operator. The partial differential equations of non-integer order derivatives have been solved by invoking Laplace and Fourier sine transforms. The new analytic solutions for temperature, concentration and velocity are investigated and expressed in terms of simple elementary functions. The corresponding general solutions have been particularized with and without magnetic field and porous medium for the classical Newtonian and second grade fluids as the limiting cases of our general results. The effects of the embedded physical and geometric parameters have been depicted through graphs for velocity, temperature and concentration showing several differences and similarities on the second grade fluid flow.

2. Mathematical formulation of the problem

Assume that an incompressible, electrically conducting and fractional second grade fluid lying over an infinite rigid plate occupying the xy plane and plate is taken normally to the y axis. Initially fluid and plate both are at rest and its temperature is T_∞ (ambient fluid temperature) and concentration is C_∞. After time $t = 0^+$, the plate begins to oscillate in its own plane and induced the motion with velocity $u(0,t) = U\sin(\omega t)$ or $u(0,t) = UH(t)\cos(\omega t)$.

Meanwhile, the heat and mass transfer from the plate are raised to temperature T_w and concentration C_w near the plate. We assume that the velocity field, temperature distribution and mass concentration are the function of y and t only. Owing to such occurrence of flow, the constraint of incompressibility is identically fulfilled. Employing the usual Boussinesq approximation, we arrive at the following set of the governing boundary layer equations for the unsteady flow in fractional form as (Shah and Khan 2016; Arshad et al., 2017; Ali et al., 2012):

$$\frac{\partial^\delta u(y,t)}{\partial t^\delta} = \nu \frac{\partial^2 u(y,t)}{\partial y^2} + \frac{\alpha_1}{\rho}\frac{\partial^3 u(y,t)}{\partial y^2 \partial t} - \frac{\mu\phi}{k}\left(\frac{\alpha_1}{\mu}\frac{\partial^\delta}{\partial t^\delta} + 1\right)u(y,t)$$
$$-\frac{\sigma B_0^2}{\rho}u(y,t) + g\beta_C(C(y,t) - C_\infty) + g\beta_T(T(y,t) - T_\infty)$$
$$, \; y,t > 0, \quad (1)$$
$$\frac{1}{k}\frac{\partial^\delta T(y,t)}{\partial t^\delta} = \frac{1}{\rho C_p}\frac{\partial^2 T(y,t)}{\partial y^2}, \quad y,t > 0, \quad (2)$$
$$\frac{1}{D}\frac{\partial^\delta C(y,t)}{\partial t^\delta} = \frac{\partial^2 C(y,t)}{\partial y^2}, \quad y,t > 0. \quad (3)$$

For developing the set of governing boundary layer Eqs. 1-3 with time-fractional derivatives, we replace the time derivative of order one with the Caputo–Fabrizio time-fractional derivative of order $0 \le \delta \le 1$. While the Caputo–Fabrizio time-fractional operator is defined as in previously published papers (Caputo and Fabrizio, 2015):

$$D_t^\delta T(y,t) = \frac{M(\delta)}{1-\delta}\int_0^t \exp\left(\frac{-\delta(t-\tau)}{1-\delta}\right)T'(\tau)d\tau,$$
$$0 \le \delta \le 1. \quad (4)$$

Here, $M(\delta)$ is a normalization function like $M(0) = M(1) = 1$. Subject to the initial and boundary conditions with no assumption of slippage between plate and fluid are

$$u(0,t) = UH(t)cos(\omega t) \; or \; Usin(\omega t), \; T(0,t) = T_w,$$
$$C(0,t) = C_w, t > t_0, \quad (5)$$
$$u(y,0) = 0, \; T(y,0) = 0, \; C(y,0) = 0, y > 0, \quad (6)$$
$$u(y,t) \to 0, \; T(y,t) \to T_\infty, \; C(y,t) \to C_\infty, \; y \to \infty, t > 0. \quad (7)$$

Implementing the following dimensionless quantities into Eqs. 1-3 and 5-7 and dropping the star notation for simplicity as:

$$t^* = \frac{U_0^2 t}{\nu}, \; y^* = \frac{U_0 y}{\nu}, \; u^* = \frac{u}{U_0}, \; C = \frac{C - C_\infty}{C_w - C_\infty},$$
$$T = \frac{T - T_\infty}{T_w - T_\infty}, \; \Phi = \frac{\mu\nu\phi}{U_0^3 k\rho}, \; \lambda = \frac{\alpha_1 U_0^2}{\mu\nu}, M = \frac{\nu\sigma B_0^2}{U_0^3\rho},$$
$$G_r = \frac{\nu g\beta_T(T_w - T_\infty)}{U_0^3}, P_r = \frac{\mu C_p}{k}, \; G_m = \frac{\nu g\beta_C(C_w - C_\infty)}{U_0^3}, \; S_c = \frac{\nu}{D}. \quad (8)$$

Under simplification, we arrive at the dimensionless governing partial differential equations in fractionalized form expressed below:

$$\frac{\partial^\delta u(y,t)}{\partial t^\delta} = \frac{\partial^2 u(y,t)}{\partial y^2}\left(1 + \lambda\frac{\partial^\delta}{\partial t^\delta}\right) + G_r T(y,t) + G_m C(y,t) -$$
$$Mu(y,t) - \Phi\left(1 + \lambda\frac{\partial^\delta}{\partial t^\delta}\right)u(y,t), \quad (9)$$
$$\frac{\partial^\delta T(y,t)}{\partial t^\delta} = \frac{1}{P_r}\frac{\partial^2 T(y,t)}{\partial y^2}, \quad y,t > 0, \quad (10)$$
$$\frac{\partial^\delta C(y,t)}{\partial t^\delta} = \frac{1}{S_c}\frac{\partial^2 C(y,t)}{\partial y^2}, \quad y,t > 0. \quad (11)$$

The suitable imposed conditions are

$$u(0,t) = u(0,t) = UH(t)cos(\omega t) \; or \; Usin(\omega t), \; T(0,t) = t, \; C(0,t) = t, \; t > 0, \quad (12)$$
$$u(y,0) = 0, \; T(y,0) = 0, \; C(y,0) = 0, y > 0, \quad (13)$$
$$u(y,t) \to 0, \; T(y,t) \to 0, \; C(y,t) \to 0, \; y \to \infty, t > 0. \quad (14)$$

3. Solution of the problem

3.1 Analytic solution of temperature distribution and mass concentration

Applying Fourier Sine transform (Abro et al., 2017; 2018c) on Eqs. 10-11 and keeping in mind Eqs. 12-14, we arrive at:

$$\frac{\partial^\delta T_s(\xi,t)}{\partial t^\delta} = \frac{1}{P_r}\left(-\xi^2 T_s(\xi,t) + \xi\sqrt{\frac{2}{\pi}}\,T(0,t)\right), \quad (15)$$
$$\frac{\partial^\delta C_s(\xi,t)}{\partial t^\delta} = \frac{1}{S_c}\left(-\xi^2 C_s(\xi,t) + \xi\sqrt{\frac{2}{\pi}}\,C(0,t)\right). \quad (16)$$

Employing Laplace transform on Eqs. 15-16 and 12-14, we get:

$$\bar{T}_s(\xi,s) = \sqrt{\frac{2}{\pi}}\frac{\xi(s+\Re_1)}{s^2(P_r\Re_0+\xi^2)(s+\Re_2)}, \quad (17)$$
$$\bar{C}_s(\xi,s) = \sqrt{\frac{2}{\pi}}\frac{\xi(s+\Re_1)}{s^2(S_c\Re_0+\xi^2)(s+\Re_3)}, \quad (18)$$

where,

$$\Re_0 = \frac{1}{1-\delta}, \Re_1 = \delta\Re_0, \Re_2 = \frac{\xi^2\delta\Re_0}{P_r\Re_0+\xi^2} \text{ and } \Re_3 = \frac{\xi^2\delta\Re_0}{S_c\Re_0+\xi^2}.$$

Inverting Eqs. 17-18 by means of Fourier Sine transform and writing Eqs. 17-18 into suitable equivalent expressions, we obtain:

$$\bar{T}(y,s) = \frac{2}{\pi}\int_0^\infty \frac{sin(y\xi)}{\xi}\left[\frac{1}{s^2} - \frac{P_r\Re_0}{(P_r\Re_0+\xi^2)s(s+\Re_2)}\right]d\xi, \quad (19)$$
$$\bar{C}(y,s) = \frac{2}{\pi}\int_0^\infty \frac{sin(y\xi)}{\xi}\left[\frac{1}{s^2} - \frac{S_c\Re_0}{(S_c\Re_0+\xi^2)s(s+\Re_3)}\right]d\xi. \quad (20)$$

Applying inverse Laplace transform and a fact of integral $\int_0^\infty \frac{sin(y\xi)}{\xi}d\xi = \frac{\pi}{2}, y > 0$ on Eqs. 19-20, we obtain final expressions for temperature distribution and mass concentration in terms of convolution theorem as:

$$T(y,t) = t + \frac{2\Re_2\Re_4}{\pi}\int_0^\infty\int_0^t \frac{sin(y\xi)}{\xi}(t-z)e^{\Re_2 t}dz\,d\xi, \quad (21)$$
$$C(y,t) = t + \frac{2\Re_3\Re_5}{\pi}\int_0^\infty\int_0^t \frac{sin(y\xi)}{\xi}(t-z)e^{\Re_3 t}dz\,d\xi. \quad (22)$$

where,

$$\Re_4 = \frac{P_r\Re_0}{(P_r\Re_0+\xi^2)} \text{ and } \Re_5 = \frac{S_c\Re_0}{(S_c\Re_0+\xi^2)}.$$

3.2 Analytic solution of velocity profile

Case-I: For Cosine oscillations

Applying Fourier Sine transform on Eq. 9 and keeping in mind Eqs. 12-14, we arrive at:

$$\frac{\partial^\delta u_s(\xi,t)}{\partial t^\delta} = \left(-\xi^2 u_s(\xi,t) + \xi\sqrt{\frac{2}{\pi}}\,u(0,t)\right)\left(1 + \lambda\frac{\partial^\delta}{\partial t^\delta}\right) -$$
$$\Phi\left(1 + \lambda\frac{\partial^\delta}{\partial t^\delta}\right)u_s(\xi,t) - Mu_s(\xi,t) + G_r T_s(\xi,t)$$
$$+G_m C_s(\xi,t) \quad (23)$$

Employing Laplace transform on Eq. 23 and Eqs. 12-14, we get:

$$\overline{u_s}(\xi,s)=U\xi\sqrt{\frac{2}{\pi}}\frac{s(s+\delta\Re_0)}{(s\Re_6+\Re_7)(s^2+\omega^2)}+\lambda U\xi\sqrt{\frac{2}{\pi}}\frac{\Re_0 s^2}{(s\Re_6+\Re_7)(s^2+\omega^2)}+\frac{G_r\bar{T}_s(\xi,s)(s+\delta\Re_0)}{(s\Re_6+\Re_7)}+\frac{G_m\bar{C}_s(\xi,s)(s+\delta\Re_0)}{(s\Re_6+\Re_7)}, \quad (24)$$

where,

$\Re_6=\Re_0+\Re_0\lambda\xi^2+\xi^2+\Phi+\Phi\Re_0\lambda+M$ and $\Re_7=\delta\Re_0(\xi^2+\Phi+M)$.

Now inverting Eq. 24 by means of Fourier Sine transform and writing it into suitable equivalent expressions, we obtain:

$$\bar{u}(y,s)=\frac{2U}{\pi}\int_0^\infty\frac{sin(y\xi)}{\xi}\left\{\frac{s}{(s^2+\omega^2)}-\frac{s(\Re_6-\xi^2)(s+\Re_8)}{\Re_6(s^2+\omega^2)(s+\Re_9)}\right\}d\xi+\frac{2U\Re_0\lambda}{\pi}\int_0^\infty\frac{\xi sin(y\xi)}{\xi}\times\frac{s^2}{(s\Re_6+\Re_7)(s^2+\omega^2)}d\xi+\frac{2G_r}{\pi}\int_0^\infty\frac{\xi sin(y\xi)}{(P_r\Re_0+\xi^2)}\frac{(s+\delta\Re_0)^2}{s^2(s+\Re_2)(s\Re_6+\Re_2)}d\xi+\frac{2G_m}{\pi}\times\int_0^\infty\frac{\xi sin(y\xi)}{(S_c\Re_0+\xi^2)}\frac{(s+\delta\Re_0)^2}{s^2(s+\Re_3)(s\Re_6+\Re_3)}d\xi, \quad (25)$$

where,

$\Re_8=\frac{(\Re_7-\delta\Re_0\xi^2)}{\Re_6-\xi^2}$ and $\Re_9=\frac{\Re_7}{\Re_6}$.

Applying inverse Laplace transform and a fact of integral $\int_0^\infty\frac{sin(y\xi)}{\xi}d\xi=\frac{\pi}{2}$ on Eq. 25, we obtain final expressions for velocity field and mass concentration in terms of convolution theorem as

$$u(y,t)_{Cosine}=UH(t)\,cos(\omega t)+\frac{2UH(t)(\Re_6-\xi^2)(\Re_9-\Re_8)}{\pi\Re_6}\int_0^\infty\int_0^t\frac{\xi sin(y\xi)}{\xi}\frac{cos\,\omega(t-\tau)}{e^{\Re_9 t}}d\xi\,d\tau+\frac{2\lambda U\Re_0\Re_9}{\pi\Re_6}\int_0^\infty\int_0^t\frac{\xi sin(y\xi)}{\xi}\frac{cos\,\omega(t-\tau)}{e^{\Re_9 t}}d\xi\,d\tau+\frac{2G_r}{\pi}\int_0^\infty\int_0^t\frac{\xi sin(y\xi)}{(P_r\Re_0+\xi^2)}\left\{\frac{(\delta\Re_0)^2 t}{\Re_6 e^{\Re_2(t-\tau)}}+\frac{\Re_{10}}{(\Re_6\Re_7)^2e^{\Re_2(t-\tau)+\Re_9 t}}+\frac{\Re_{11}}{\Re_7{}^2e^{\Re_2(t-\tau)}}\right\}d\xi\,d\tau+\frac{2G_m}{\pi}\int_0^\infty\int_0^t\frac{\xi sin(y\xi)}{(S_c\Re_0+\xi^2)}\left\{\frac{(\delta\Re_0)^2 t}{\Re_6 e^{\Re_3(t-\tau)}}+\frac{\Re_{10}}{(\Re_6\Re_7)^2e^{\Re_3(t-\tau)+\Re_9 t}}+\frac{\Re_{11}}{\Re_7{}^2e^{\Re_3(t-\tau)}}\right\}d\xi\,d\tau, \quad (26)$$

where,

$\Re_{10}=(\delta\Re_0\Re_7)^2-2\delta\Re_0\Re_6\Re_7+\Re_7{}^2$,
$\Re_{11}=2\delta\Re_0\Re_7-\delta\Re_0(\delta\Re_0)^2\Re_7$.

Case-II: For Sine oscillation

By invoking similar algorithm, we investigated the velocity field for sine oscillation from Eq. 9, we obtain:

$$u(y,t)_{Sine}=U\,sin(\omega t)+\frac{2U(\Re_6-\xi^2)(\Re_9-\Re_8)}{\pi\Re_6}\int_0^\infty\int_0^t\frac{\xi sin(y\xi)}{\xi}\frac{sin\,\omega(t-\tau)}{e^{\Re_9 t}}d\xi\,d\tau+\frac{2\lambda U\Re_0\Re_9}{\pi\Re_6}\int_0^\infty\int_0^t\frac{\xi sin(y\xi)}{\xi}\frac{sin\,\omega(t-\tau)}{e^{\Re_9 t}}d\xi\,d\tau+\frac{2G_r}{\pi}\int_0^\infty\int_0^t\frac{\xi sin(y\xi)}{(P_r\Re_0+\xi^2)}\left\{\frac{(\delta\Re_0)^2 t}{\Re_6 e^{\Re_2(t-\tau)}}+\frac{\Re_{10}}{(\Re_6\Re_7)^2e^{\Re_2(t-\tau)+\Re_9 t}}+\frac{\Re_{11}}{\Re_7{}^2e^{\Re_2(t-\tau)}}\right\}d\xi\,d\tau+\frac{2G_m}{\pi}\int_0^\infty\int_0^t\frac{\xi sin(y\xi)}{\left(S_c\Re_0+\xi^2\right)}\left\{\frac{(\delta\Re_0)^2 t}{\Re_6 e^{\Re_3(t-\tau)}}+\frac{\Re_{10}}{(\Re_6\Re_7)^2e^{\Re_3(t-\tau)+\Re_9 t}}+\frac{\Re_{11}}{\Re_7{}^2e^{\Re_3(t-\tau)}}\right\}d\xi\,d\tau. \quad (27)$$

4. Special solutions

4.1 Velocity field of fractional second grade fluid without magnetic field with porous medium

Letting $M=0,\Phi\neq 0,\lambda\neq 0$ in the Eq. 26 and Eq. 27, we reduced the general analytical solutions for Caputo-Fabrizio fractional second grade fluid in the absence of magnetic field with porous medium for sine and cosine oscillations as

$$u(y,t)_{Cosine}=UH(t)\cos(\omega t)+\frac{2UH(t)(\Re_{12}-\xi^2)(\Re_{15}-\Re_{14})}{\pi\Re_{12}}\int_0^\infty\int_0^t\frac{\xi\sin(y\xi)cos\omega(t-\tau)}{e^{\Re_{15}t}}d\xi d\tau+\frac{2\lambda U\Re_0\Re_{15}}{\pi\Re_{12}}\int_0^\infty\int_0^t\frac{\xi\sin(y\xi)}{\xi}\frac{cos\omega(t-\tau)}{e^{\Re_{15}t}}d\xi d\tau+\frac{2G_r}{\pi}\int_0^\infty\int_0^t\frac{\xi\sin(y\xi)}{(P_r\Re_0+\xi^2)}\left\{\frac{(\delta\Re_0)^2t}{\Re_{12}e^{\Re_2(t-\tau)}}+\frac{\Re_{16}}{(\Re_{12}\Re_{13})^2e^{\Re_2(t-\tau)+\Re_{15}t}}+\frac{\Re_{17}}{\Re_{13}^2e^{\Re_2(t-\tau)}}\right\}d\xi d\tau+\frac{2G_m}{\pi}\int_0^\infty\int_0^t\frac{\xi\sin(y\xi)}{(S_c\Re_0+\xi^2)}\times\left\{\frac{(\delta\Re_0)^2t}{\Re_{12}e^{\Re_3(t-\tau)}}+\frac{\Re_{10}}{(\Re_{12}\Re_{13})^2e^{\Re_3(t-\tau)+\Re_{15}t}}+\frac{\Re_{17}}{\Re_{13}^2e^{\Re_3(t-\tau)}}\right\}d\xi d\tau, \quad (28)$$

$$u(y,t)_{Sine}=U\sin(\omega t)+\frac{2U(\Re_{12}-\xi^2)(\Re_{15}-\Re_{14})}{\pi\Re_{12}}\int_0^\infty\int_0^t\frac{\xi\sin(y\xi)sin\omega(t-\tau)}{e^{\Re_{15}t}}d\xi d\tau+\frac{2\lambda U\Re_0\Re_{15}}{\pi\Re_{12}}\int_0^\infty\int_0^t\frac{\sin(y\xi)}{\xi}\frac{sin\omega(t-\tau)}{e^{\Re_{15}t}}d\xi d\tau+\frac{2G_r}{\pi}\int_0^\infty\int_0^t\frac{\xi\sin(y\xi)}{(P_r\Re_0+\xi^2)}\left\{\frac{(\delta\Re_0)^2t}{\Re_{12}e^{\Re_2(t-\tau)}}+\frac{\Re_{16}}{(\Re_{12}\Re_{13})^2e^{\Re_2(t-\tau)+\Re_{15}t}}+\frac{\Re_{17}}{\Re_{13}^2e^{\Re_2(t-\tau)}}\right\}d\xi d\tau+\frac{2G_m}{\pi}\int_0^\infty\int_0^t\frac{\xi\sin(y\xi)}{(S_c\Re_0+\xi^2)}\times\left\{\frac{(\delta\Re_0)^2t}{\Re_{12}e^{\Re_3(t-\tau)}}+\frac{\Re_{10}}{(\Re_{12}\Re_{13})^2e^{\Re_3(t-\tau)+\Re_{15}t}}+\frac{\Re_{17}}{\Re_{13}^2e^{\Re_3(t-\tau)}}\right\}d\xi d\tau. \quad (29)$$

where,

$\Re_{12}=\Re_0+\Re_0\lambda\xi^2+\xi^2+\Phi+\Phi\Re_0\lambda$, $\Re_{13}=\Re_1(\xi^2+\Phi)$, $\Re_{14}=\frac{(\Re_{13}-\Re_1\xi^2)}{(\Re_{12}-\xi^2)}$, $\Re_{15}=\frac{\Re_{13}}{\Re_{12}}$,
$\Re_{16}=(\Re_1\Re_{13})^2-2\Re_1\Re_{12}\Re_{13}+(\Re_{13})^2$ *and* $\Re_{17}=2\Re_1\Re_{13}-(\Re_1)^3$.

4.2 Velocity field of fractional second grade fluid without porous medium with magnetic field

Employing $\Phi=0,M\neq 0,\lambda\neq 0$ in the Eq. 26 and Eq. 27, we reduced the general analytical solutions for Caputo-Fabrizio fractional second grade fluid in the absence of porous medium with magnetic field for sine and cosine oscillations as

$$u(y,t)_{Cosine}=UH(t)\cos(\omega t)+\frac{2UH(t)(\Re_{18}-\xi^2)(\Re_{21}-\Re_{20})}{\pi\Re_{18}}\int_0^\infty\int_0^t\frac{\xi\sin(y\xi)cos\omega(t-\tau)}{e^{\Re_{21}t}}d\xi d\tau+\frac{2\lambda U\Re_0\Re_{21}}{\pi\Re_{18}}\int_0^\infty\int_0^t\frac{\xi\sin(y\xi)}{\xi}\frac{cos\omega(t-\tau)}{e^{\Re_{21}t}}d\xi d\tau+\frac{2G_r}{\pi}\int_0^\infty\int_0^t\frac{\xi\sin(y\xi)}{(P_r\Re_0+\xi^2)}\left\{\frac{(\delta\Re_0)^2t}{\Re_{18}e^{\Re_2(t-\tau)}}+\frac{\Re_{22}}{(\Re_{18}\Re_{19})^2e^{\Re_2(t-\tau)+\Re_{21}t}}+\right.$$

$$\left.\frac{\Re_{23}}{\Re_{19}^2 e^{\Re_2(t-\tau)}}\right\} d\xi d\tau + \frac{2G_m}{\pi}\int_0^\infty\int_0^t \frac{\xi\sin(y\xi)}{(S_c\Re_0+\xi^2)} \times \left\{\frac{(\delta\Re_0)^2 t}{\Re_{18}e^{\Re_3(t-\tau)}} + \frac{\Re_{22}}{(\Re_{18}\Re_{19})^2 e^{\Re_3(t-\tau)+\Re_{21}t}} + \frac{\Re_{23}}{\Re_{19}^2 e^{\Re_3(t-\tau)}}\right\} d\xi d\tau, \qquad (30)$$

$$u(y,t)_{Sine} = U\sin(\omega t) + \frac{2U(\Re_{18}-\xi^2)(\Re_{21}-\Re_{20})}{\pi\Re_{18}}\int_0^\infty\int_0^t \frac{\xi\sin(y\xi)sin\omega(t-\tau)}{e^{\Re_{21}t}} d\xi d\tau + \frac{2\lambda U\Re_0\Re_{21}}{\pi\Re_{18}}\int_0^\infty\int_0^t \frac{\xi\sin(y\xi)}{\xi}\frac{sin\omega(t-\tau)}{e^{\Re_{21}t}} d\xi d\tau + \frac{2G_r}{\pi}\int_0^\infty\int_0^t \frac{\xi\sin(y\xi)}{(P_r\Re_0+\xi^2)}\left\{\frac{(\delta\Re_0)^2 t}{\Re_{18}e^{\Re_2(t-\tau)}} + \frac{\Re_{22}}{(\Re_{18}\Re_{19})^2 e^{\Re_2(t-\tau)+\Re_{21}t}} + \frac{\Re_{23}}{\Re_{19}^2 e^{\Re_2(t-\tau)}}\right\} d\xi d\tau + \frac{2G_m}{\pi}\int_0^\infty\int_0^t \frac{\xi\sin(y\xi)}{(S_c\Re_0+\xi^2)} \times \left\{\frac{(\delta\Re_0)^2 t}{\Re_{18}e^{\Re_3(t-\tau)}} + \frac{\Re_{22}}{(\Re_{18}\Re_{19})^2 e^{\Re_3(t-\tau)+\Re_{21}t}} + \frac{\Re_{23}}{\Re_{19}^2 e^{\Re_3(t-\tau)}}\right\} d\xi d\tau, \qquad (31)$$

where,

$\Re_{18} = \Re_0 + \Re_0\lambda\xi^2 + \xi^2 + M, \Re_{19} = \Re_1(\xi^2+M), \Re_{20} = \frac{(\Re_{19}-\alpha\Re_0\xi^2)}{(\Re_{18}-\xi^2)}, \Re_{21} = \frac{\Re_{19}}{\Re_{18}},$
$\Re_{22} = (\Re_1\Re_{19})^2 - 2\Re_1\Re_{18}\Re_{19} + (\Re_{19})^2$ and $\Re_{23} = 2\Re_1\Re_{19} - (\Re_1)^3\Re_{19}.$

4.3 Velocity field of fractional Newtonian fluid with magnetic field and porous medium

Letting $\lambda = 0, M \neq 0, \Phi \neq 0$ in the Eq. 26 and Eq. 27, we obtain the general analytical solutions for Caputo-Fabrizio fractional Newtonian fluid in the presence of porous medium and magnetic field for sine and cosine oscillations as:

$$u(y,t)_{Cosine} = UH(t)\,cos(\omega t) - \frac{2UH(t)(\Re_{24}-\xi^2)(\Re_{26}-\Re_{25})}{\pi\Re_{24}}\int_0^\infty\int_0^t \frac{sin(y\xi)cos\omega(t-\tau)}{\xi e^{\Re_{26}t}} d\xi d\tau + \frac{2G_r}{\pi}\int_0^\infty\int_0^t \frac{\xi sin(y\xi)}{(P_r\Re_0+\xi^2)}\left\{\frac{(\delta\Re_0)^2 t}{\Re_{24}e^{\Re_2(t-\tau)}} + \frac{\Re_{27}}{(\Re_{24}\Re_7)^2 e^{\Re_2(t-\tau)+\Re_{26}t}} + \frac{\Re_{28}}{\Re_7^2 e^{\Re_2(t-\tau)}}\right\} d\xi d\tau + \frac{2G_m}{\pi}\int_0^\infty\int_0^t \frac{\xi\, sin(y\xi)}{(S_c\Re_0+\xi^2)}\left\{\frac{(\delta\Re_0)^2 t}{\Re_{24}e^{\Re_3(t-\tau)}} + \frac{\Re_{27}}{(\Re_{24}\Re_7)^2 e^{\Re_3(t-\tau)+\Re_{26}t}} + \frac{\Re_{28}}{\Re_7^2 e^{\Re_3(t-\tau)}}\right\} d\xi d\tau, \qquad (32)$$

$$u(y,t)_{Sine} = U\,sin(\omega t) - \frac{2U(\Re_{24}-\xi^2)(\Re_{26}-\Re_{25})}{\pi\Re_{24}}\int_0^\infty\int_0^t \frac{sin(y\xi)sin\omega(t-\tau)}{\xi e^{\Re_{26}t}} d\xi d\tau + \frac{2G_r}{\pi}\int_0^\infty\int_0^t \frac{\xi sin(y\xi)}{(P_r\Re_0+\xi^2)}\left\{\frac{(\delta\Re_0)^2 t}{\Re_{24}e^{\Re_2(t-\tau)}} + \frac{\Re_{27}}{(\Re_{24}\Re_7)^2 e^{\Re_2(t-\tau)+\Re_{26}t}} + \frac{\Re_{28}}{\Re_7^2 e^{\Re_2(t-\tau)}}\right\} d\xi d\tau + \frac{2G_m}{\pi}\int_0^\infty\int_0^t \frac{\xi sin(y\xi)}{(S_c\Re_0+\xi^2)}\left\{\frac{(\delta\Re_0)^2 t}{\Re_{24}e^{\Re_3(t-\tau)}} + \frac{\Re_{27}}{(\Re_{24}\Re_7)^2 e^{\Re_3(t-\tau)+\Re_{26}t}} + \frac{\Re_{28}}{\Re_7^2 e^{\Re_3(t-\tau)}}\right\} d\xi d\tau. \qquad (33)$$

where,

$\Re_{24} = \Re_0 + \xi^2 + \Phi + \Phi\Re_0\lambda + M, \Re_7 = \alpha\Re_0(\xi^2 + \Phi + M), \Re_{25} = \frac{(\Re_7 - \Re_1\xi^2)}{(\Re_{24}-\xi^2)}, \Re_{26} = \frac{\Re_7}{\Re_{24}},$
$\Re_{27} = (\Re_1\Re_7)^2 - 2\Re_1\Re_{24}\Re_7 + (\Re_7)^2$ and $\Re_{28} = 2\Re_1\Re_7 - (\Re_1)^3\Re_7.$

4.4 Velocity field of fractional Newtonian fluid without magnetic field with porous medium

Letting $\lambda = 0, M = 0, \Phi \neq 0$ in the Eq. 26 and Eq. 27, we obtain the general analytical solutions for Caputo-Fabrizio fractional Newtonian fluid in the presence of porous medium and without magnetic field for sine and cosine oscillations as:

$$u(y,t)_{Cosine} = UH(t)\,cos(\omega t) - \frac{2UH(t)(\Re_{29}-\xi^2)(\Re_{31}-\Re_{30})}{\pi\Re_{29}}\int_0^\infty\int_0^t \frac{sin(y\xi)cos\omega(t-\tau)}{\xi e^{\Re_{31}t}} d\xi d\tau + \frac{2G_r}{\pi}\int_0^\infty\int_0^t \frac{\xi sin(y\xi)}{(P_r\Re_0+\xi^2)}\left\{\frac{(\delta\Re_0)^2 t}{\Re_{29}e^{\Re_2(t-\tau)}} + \frac{\Re_{32}}{(\Re_{29}\Re_{13})^2 e^{\Re_2(t-\tau)+\Re_{26}t}} + \frac{\Re_{33}}{\Re_{13}^2 e^{\Re_2(t-\tau)}}\right\} d\xi d\tau + \frac{2G_m}{\pi}\int_0^\infty\int_0^t \frac{\xi sin(y\xi)}{(S_c\Re_0+\xi^2)}\left\{\frac{(\delta\Re_0)^2 t}{\Re_{29}e^{\Re_3(t-\tau)}} + \frac{\Re_{32}}{(\Re_{29}\Re_{13})^2 e^{\Re_3(t-\tau)+\Re_{26}t}} + \frac{\Re_{33}}{\Re_{13}^2 e^{\Re_3(t-\tau)}}\right\} d\xi d\tau, \qquad (34)$$

$$u(y,t)_{Sine} = U\,\text{sine}(\omega t) - \frac{2U(\Re_{29}-\xi^2)(\Re_{31}-\Re_{30})}{\pi\Re_{29}}\int_0^\infty\int_0^t \frac{\sin(y\xi)sin\omega(t-\tau)}{\xi e^{\Re_{31}t}} d\xi d\tau + \frac{2G_r}{\pi}\int_0^\infty\int_0^t \frac{\xi\sin(y\xi)}{(P_r\Re_0+\xi^2)}\left\{\frac{(\delta\Re_0)^2 t}{\Re_{29}e^{\Re_2(t-\tau)}} + \frac{\Re_{32}}{(\Re_{29}\Re_{13})^2 e^{\Re_2(t-\tau)+\Re_{26}t}} + \frac{\Re_{33}}{\Re_{13}^2 e^{\Re_2(t-\tau)}}\right\} d\xi d\tau + \frac{2G_m}{\pi}\int_0^\infty\int_0^t \frac{\xi\sin(y\xi)}{(S_c\Re_0+\xi^2)}\left\{\frac{(\delta\Re_0)^2 t}{\Re_{29}e^{\Re_3(t-\tau)}} + \frac{\Re_{32}}{(\Re_{29}\Re_{13})^2 e^{\Re_3(t-\tau)+\Re_{26}t}} + \frac{\Re_{33}}{\Re_{13}^2 e^{\Re_3(t-\tau)}}\right\} d\xi d\tau. \qquad (35)$$

where,

$\Re_{29} = \Re_0 + \xi^2 + \Phi + \Phi\Re_0\lambda, \Re_{13} = \delta\Re_0(\xi^2 + \Phi +), \Re_{30} = \frac{(\Re_{13} - \Re_1\xi^2)}{(\Re_{29}-\xi^2)}, \Re_{31} = \frac{\Re_{13}}{\Re_{29}}, \Re_{32} = (\Re_1\Re_{13})^2 - 2\Re_1\Re_{29}\Re_{13} + (\Re_{13})^2$ and $\Re_{33} = 2\Re_1\Re_{13} - (\Re_1)^3\Re_{13}$

4.5 Velocity field of fractional Newtonian fluid with magnetic field with without porous medium

Letting $\lambda = 0, M \neq 0, \Phi = 0$ in the Eq. 26 and Eq. 27, we obtain the general analytical solutions for Caputo-Fabrizio fractional Newtonian fluid in the absence of porous medium and with magnetic field for sine and cosine oscillations as:

$$u(y,t)_{Cosine} = UH(t)\cos(\omega t) - \frac{2UH(t)(\Re_{34}-\xi^2)(\Re_{36}-\Re_{35})}{\pi\Re_{34}}\int_0^\infty\int_0^t \frac{\sin(y\xi)cos\omega(t-\tau)}{\xi e^{\Re_{31}t}} d\xi d\tau + \frac{2G_r}{\pi}\int_0^\infty\int_0^t \frac{\xi\sin(y\xi)}{(P_r\Re_0+\xi^2)}\left\{\frac{(\delta\Re_0)^2 t}{\Re_{34}e^{\Re_2(t-\tau)}} + \frac{\Re_{37}}{(\Re_{34}\Re_{19})^2 e^{\Re_2(t-\tau)+\Re_{36}t}} + \frac{\Re_{38}}{\Re_{19}^2 e^{\Re_2(t-\tau)}}\right\} d\xi d\tau + \frac{2G_m}{\pi}\int_0^\infty\int_0^t \frac{\xi\sin(y\xi)}{(S_c\Re_0+\xi^2)}\left\{\frac{(\delta\Re_0)^2 t}{\Re_{34}e^{\Re_3(t-\tau)}} + \frac{\Re_{37}}{(\Re_{34}\Re_{19})^2 e^{\Re_3(t-\tau)+\Re_{36}t}} + \frac{\Re_{38}}{\Re_{19}^2 e^{\Re_3(t-\tau)}}\right\} d\xi d\tau, \qquad (36)$$

$$u(y,t)_{Sine} = U\sin(\omega t) - \frac{2U(\Re_{34}-\xi^2)(\Re_{36}-\Re_{35})}{\pi\Re_{34}}\int_0^\infty\int_0^t \frac{\sin(y\xi)sin\omega(t-\tau)}{\xi e^{\Re_{31}t}} d\xi d\tau + \frac{2G_r}{\pi}\int_0^\infty\int_0^t \frac{\xi\sin(y\xi)}{(P_r\Re_0+\xi^2)}\left\{\frac{(\delta\Re_0)^2 t}{\Re_{34}e^{\Re_2(t-\tau)}} + \frac{\Re_{37}}{(\Re_{34}\Re_{19})^2 e^{\Re_2(t-\tau)+\Re_{36}t}} + \frac{\Re_{38}}{\Re_{19}^2 e^{\Re_2(t-\tau)}}\right\} d\xi d\tau + \frac{2G_m}{\pi}\int_0^\infty\int_0^t \frac{\xi\sin(y\xi)}{(S_c\Re_0+\xi^2)}\left\{\frac{(\delta\Re_0)^2 t}{\Re_{34}e^{\Re_3(t-\tau)}} + \frac{\Re_{37}}{(\Re_{34}\Re_{19})^2 e^{\Re_3(t-\tau)+\Re_{36}t}} + \frac{\Re_{38}}{\Re_{19}^2 e^{\Re_3(t-\tau)}}\right\} d\xi d\tau. \qquad (37)$$

where,

$\Re_{34} = \Re_0 + \xi^2 + M, \Re_{19} = \Re_1(\xi^2+M), \Re_{35} = \frac{(\Re_{19}-\Re_1\xi^2)}{(\Re_{34}-\xi^2)}, \Re_{36} = \frac{\Re_{19}}{\Re_{34}},$
$\Re_{37} = (\Re_1)^2 - 2\Re_1\Re_{34}\Re_{19} + {\Re_{19}}^2, \Re_{38} = 2\Re_1\Re_{19} - (\Re_1)^3\Re_{19}.$

However, letting $\delta = 1$ in the Eq. 26 and Eq. 27, we obtain the general analytical solutions for ordinary second grade fluid in the presence of porous medium and magnetic field for sine and cosine oscillations as well. Furthermore, the present solutions obtained by Caputo-Fabrizio fractional derivative become identical and similar solutions investigated in Shah and Khan (2016) (see Eq. 22 and Eq. 26) when $G_m = 0$ (in the absence of mass concentration), $M = 0$ (in the absence of magnetic field) and $\Phi = 0$ (in the absence of porous medium). Meanwhile, when we substitute $M = \Phi = \omega = 0$ in present solutions, our solutions can be retrieved in the absence of magnetic field and porous medium with Caputo-Fabrizio fractional operator. Such fractional solutions are investigated in literature obtained by Arshad et al. (2017) (see Eq. 47).

5. Parametric results and conclusion

This paragraph emphasizes on the numerical results and discussions for the analysis of the flow electrically conducting second grade fluid with heat and mass transfer embedded in porous medium. By invoking Laplace and Fourier sine transforms on non-integer order differential equations, the new analytic solutions for temperature, concentration and velocity are investigated. The general solutions are particularized with and without magnetic field and porous medium for the classical Newtonian and second grade fluids as the special solutions. The salient impacts of distinct parameters are reported graphically that show several physical aspects on the fluid flow. It is worth pointed out that the main novelty of this work is to check the influences of the analytic solutions on the graphical comparison for four types of models namely (i) Caputo-Fabrizio fractional solutions for second grade fluid with and without magnetic field and porous medium, (ii) Ordinary solutions for second grade fluid with and without magnetic field and porous medium (iii) Caputo-Fabrizio fractional solutions for Newtonian (viscous) fluid with and without magnetic field and porous medium and (iv) Ordinary solutions for Newtonian (viscous) fluid with and without magnetic field and porous medium. In brevity, the major highlights are described in context with physical aspects as enumerated below:

(i) Fig. 1 is prepared to display the impacts of Caputo-Fabrizio fractional parameter on the profile of the temperature distribution and mass concentration. It can be seen that the enhancing the values of Caputo-Fabrizio fractional parameter δ the behavior is decreasing function in terms of fractional parameter δ. This is due to the fact that diffusion penetrates deeper into the fluid, hence it causes the thickening of the concentration boundary layer as well as the thermal boundary layer.

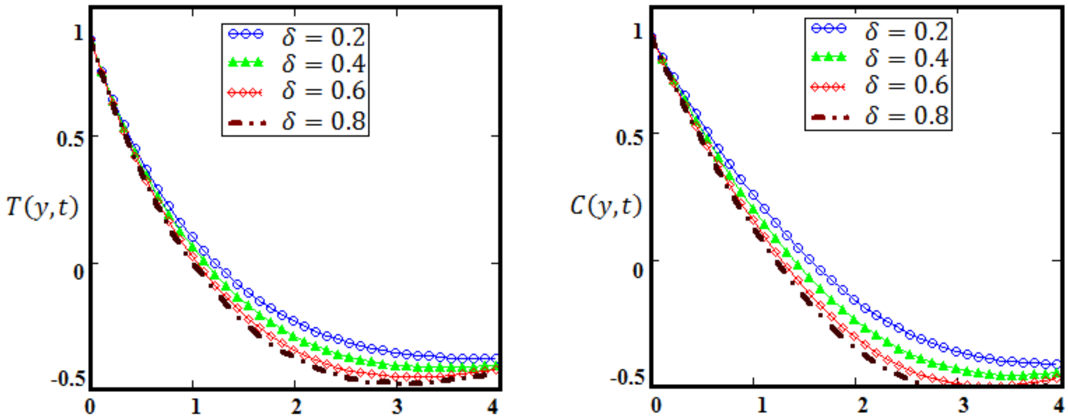

Fig. 1: Profile of temperature distribution and mass concentration for Caputo-Fabrizio fractional parameter

(ii) It is well established fact that Prandtl number Pr plays a significant role among industries and engineering systems as it depends upon quality production, such quality production can be achieved via suitable choice of Prandtl number Pr. Here Fig. 2 is depicted for the effects of Prandtl Pr and Schmidt Sc numbers on temperature distribution and mass concertation respectively. It is noted that increase in Prandtl number Pr does not give reduction in thermal layer. Even weaker conductivity depends upon larger amount of Prandtl number Pr. On the contrary, similar trend is noticed in mass concertation with respect to Schmidt number.

(iii) Fig. 3 elucidates the modified Grashof and thermal Grashof numbers on the velocity field, here we considered suitable values for $Gr = 2,4,6,8$ and thermal Grashof $Gm = 1,3,5,7$. It is noted that an increase in the velocity profile is observed when the Grashof number is increased. From physical point of view by increasing the Grashof number, heat transfer due to convection facilitates the flow velocity profile. Meanwhile, an opposite trend is observed in thermal Grashof number with similar effects as compared to the Grashof number.

(iv) Effects of magnetic field and porous medium on the velocity field are presented in Fig. 4. The presence of magnetic field exerts resistive force so called Lorentz force, as the result flow velocity reduces due to existence of magnetic field. On the contrary, it is noted that the velocity field has reciprocal behavior for the analysis of porosity. Hence, increase in porosity increases the velocity profile.

(v) It is worth pointed out from Fig. 5 that the graphical comparison for four types of models can be analyzed namely (i) Caputo-Fabrizio fractional solutions for second grade fluid with and without magnetic field and porous medium, (ii) Ordinary solutions for second grade fluid with and without magnetic field and porous medium (iii) Caputo-Fabrizio fractional solutions for Newtonian (viscous) fluid with and without magnetic field and porous medium and (iv) Ordinary solutions for Newtonian (viscous) fluid with and without magnetic field and porous medium. In this comparative analysis, an observation declares that ordinary Newtonian fluid with magnetic field and porous medium is dominant than other models. In simple words, ordinary Newtonian fluid with magnetic field and porous medium moves faster in comparison with remaining models of the interest. However, the observation for above models has been performed for the sake of simplicity of this analysis. Meanwhile, the same phenomenon can also be analyzed for the temperature and concentration distribution via ordinary and Caputo-Fabrizio fractional operator.

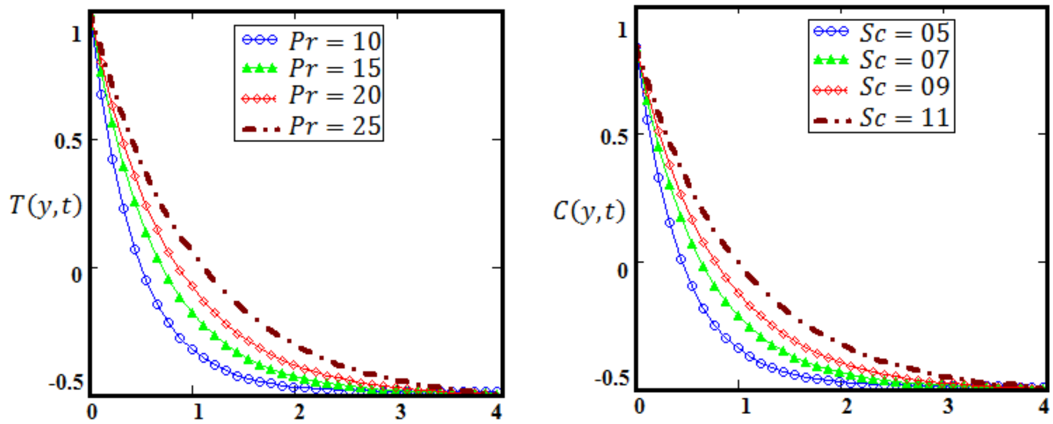

Fig. 2: Profile of temperature distribution and mass concentration for Prandtl and Schmidt numbers respectively

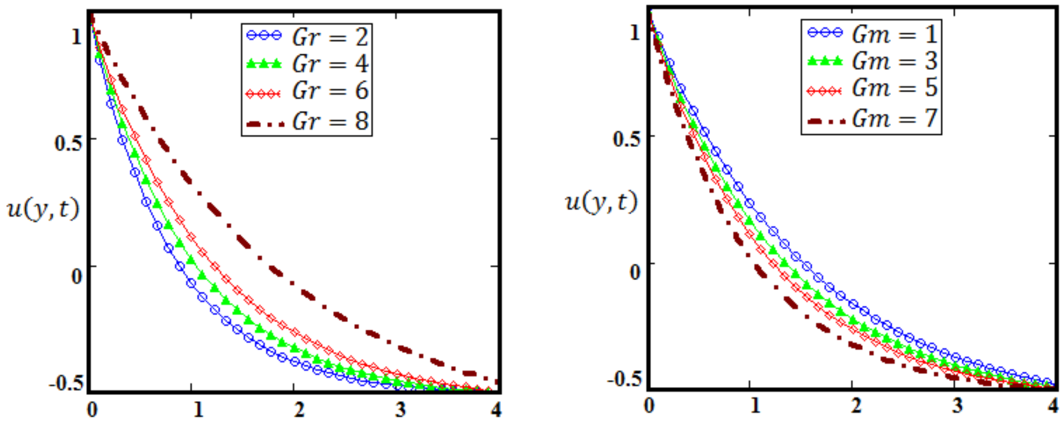

Fig. 3: Profile of velocity field for modified Grashof and thermal Grashof numbers respectively

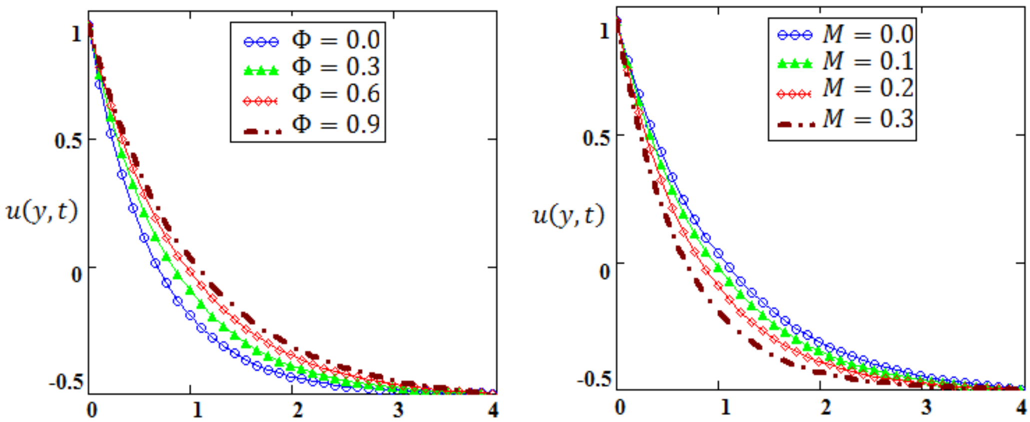

Fig. 4: Profile of velocity field for porous and magnetic field respectively

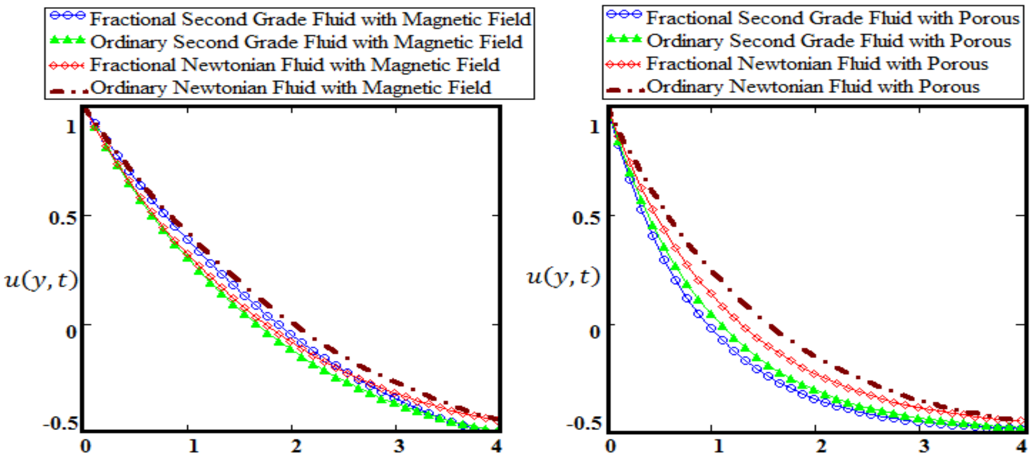

Fig. 5: Profile of velocity field comparison of fractional and ordinary models with and without magnetic field and porous medium

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